Ch 9. UNSYMMETRICAL FAULTS

Order of short circuit frequency of occurrence:

1- Line-to-ground.
2- Line-to-line.
3- Double line-to-ground.
4- Balanced three-phase.

9.1 System Representation:

* System assumptions:

1. The power system operates under balanced steady-state conditions before the fault occurs. Thus the zero-, positive-, and negative-sequence networks are uncoupled before the fault occurs. During unsymmetrical faults they are interconnected only at the fault location.
2. Prefault load current is neglected. Because of this, the positive-sequence internal voltages of all machines are equal to the prefault voltage V_F. Therefore, the prefault voltage at each bus in the positive-sequence network equals V_F.
3. Transformer winding resistances and shunt admittances are neglected.
4. Transmission-line series resistances and shunt admittances are neglected.
5. Synchronous machine armature resistance, saliency, and saturation are neglected.
6. All nonrotating impedance loads are neglected.
7. Induction motors are either neglected (especially for motors rated 50 hp or less) or represented in the same manner as synchronous machines.

* Faults at the general three-phase bus is represented using:

FIGURE 9.1 General three-phase bus (Phase domain)

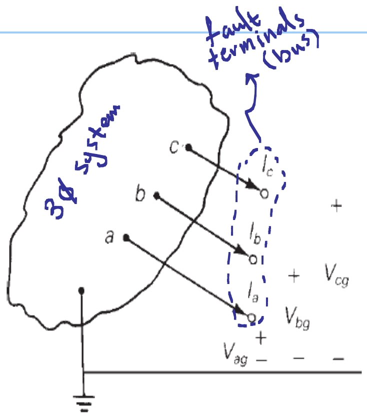
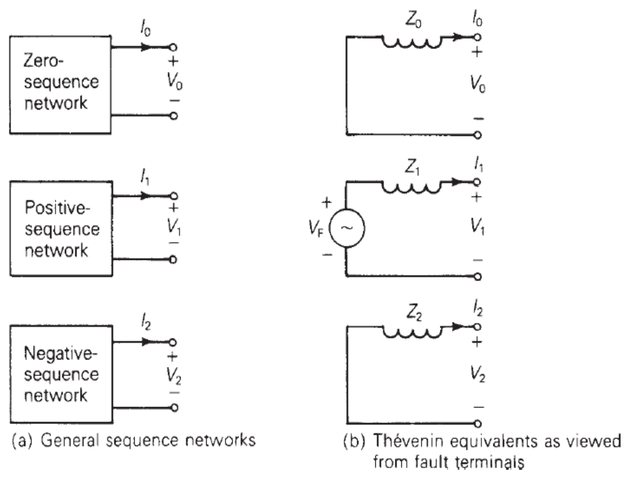


FIGURE 9.2 Sequence networks at a general three-phase bus in a balanced system

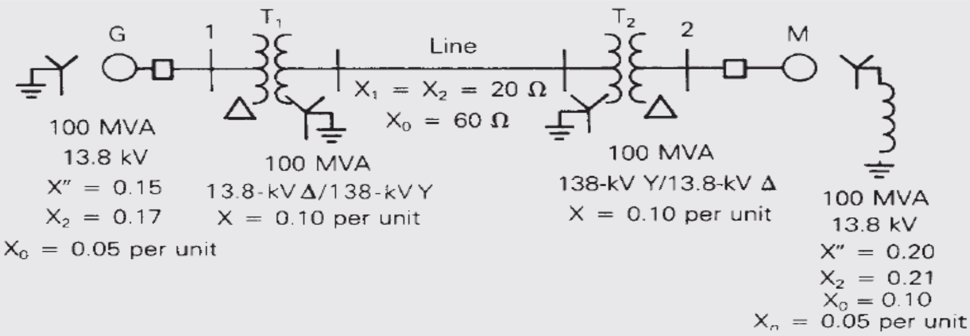


EXAMPLE 9.1 Power-system sequence networks and their Thévenin equivalents

A single-line diagram of the power system considered in Example 7.3 is shown in Figure 9.3, where negative- and zero-sequence reactances are also given. The neutrals of the generator and Δ–Y transformers are solidly grounded. The motor neutral is grounded through a reactance $X_n = 0.05$ per unit on the motor base. (a) Draw the per-unit zero-, positive-, and negative-sequence networks on a 100-MVA, 13.8-kV base in the zone of the generator. (b) Reduce the sequence networks to their Thévenin equivalents, as viewed from bus 2. Prefault voltage is $V_F = 1.05\underline{/0°}$ per unit. Prefault load current and Δ–Y transformer phase shift are neglected.

FIGURE 9.3

Single-line diagram for Example 9.1

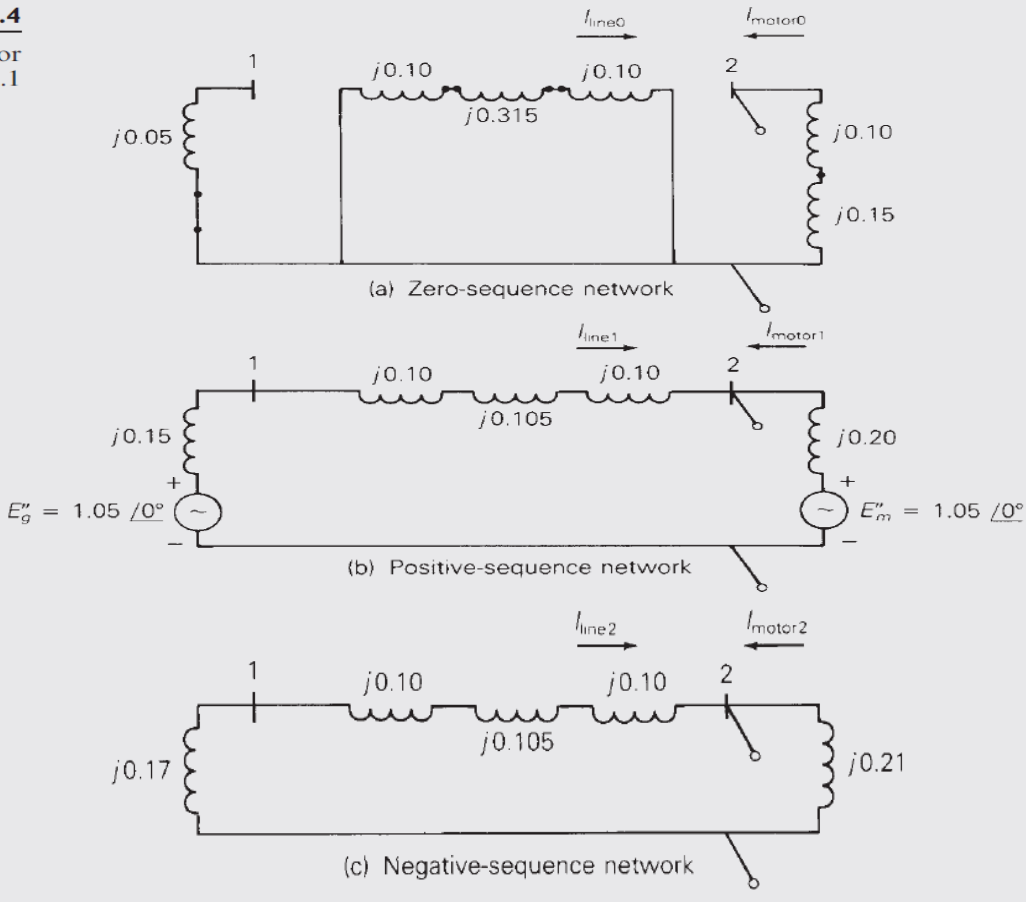


SOLUTION

a. The sequence networks are shown in Figure 9.4. The positive-sequence network is the same as that shown in Figure 7.4(a). The negative-sequence network is similar to the positive-sequence network, except that there are no sources, and negative-sequence machine reactances are shown. Δ–Y phase shifts are omitted from the positive- and negative-sequence networks for this example. In the zero-sequence network the zero-sequence generator, motor, and transmission-line reactances are shown. Since the motor neutral is grounded through a neutral reactance X_n, $3X_n$ is included in the zero-sequence motor circuit. Also, the zero-sequence Δ–Y transformer models are taken from Figure 8.19.

FIGURE 9.4

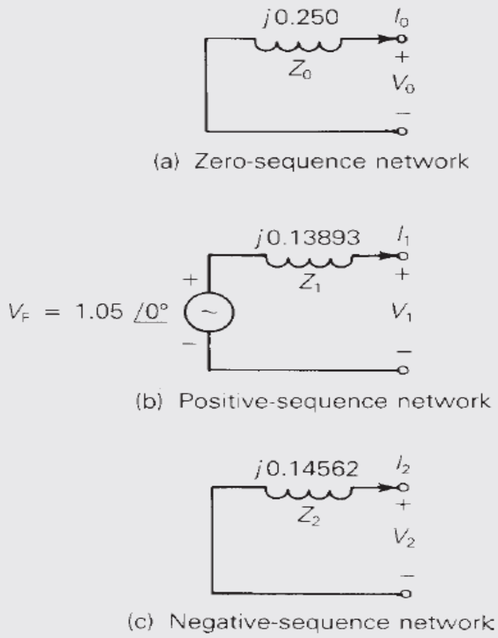
Sequence networks for Example 9.1



b. Figure 9.5 shows the sequence networks reduced to their Thévenin equivalents, as viewed from bus 2. For the positive-sequence equivalent, the Thévenin voltage source is the prefault voltage $V_F = 1.05\underline{/0°}$ per unit.

FIGURE 9.5

Thévenin equivalents of sequence networks for Example 9.1



From Figure 9.4, the positive-sequence Thévenin impedance at bus 2 is the motor impedance $j0.20$, as seen to the right of bus 2, in parallel with $j(0.15 + 0.10 + 0.105 + 0.10) = j0.455$, as seen to the left; the parallel combination is $j0.20 /\!/ j0.455 = j0.13893$ per unit. Similarly, the negative-sequence Thévenin impedance is $j0.21 /\!/ j(0.17 + 0.10 + 0.105 + 0.10) = j0.21 /\!/ j0.475 = j0.14562$ per unit. In the zero-sequence network of Figure 9.4, the Thévenin impedance at bus 2 consists only of $j(0.10 + 0.15) = j0.25$ per unit, as seen to the right of bus 2; due to the Δ connection of transformer T_2, the zero-sequence network looking to the left of bus 2 is open. ■

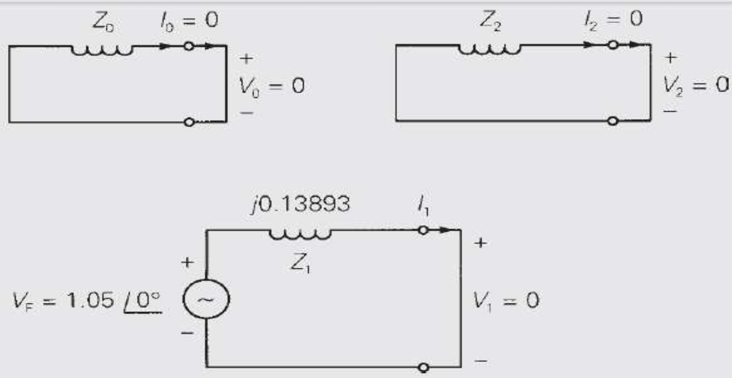
EXAMPLE 9.2 Three-phase short-circuit calculations using sequence networks

Calculate the per-unit subtransient fault currents in phases a, b, and c for a bolted three-phase-to-ground short circuit at bus 2 in Example 9.1.

SOLUTION The terminals of the positive-sequence network in Figure 9.5(b) are shorted, as shown in Figure 9.6. The positive-sequence fault current is

FIGURE 9.6

Example 9.2: Bolted three-phase-to-ground fault at bus 2



$$I_1 = \frac{V_F}{Z_1} = \frac{1.05\underline{/0°}}{j0.13893} = -j7.558 \quad \text{per unit}$$

which is the same result as obtained in part (c) of Example 7.4. Note that since subtransient machine reactances are used in Figures 9.4–9.6, the current calculated above is the positive-sequence subtransient fault current at bus 2. Also, the zero-sequence current I_0 and negative-sequence current I_2 are both zero. Therefore, the subtransient fault currents in each phase are, from (8.1.16),

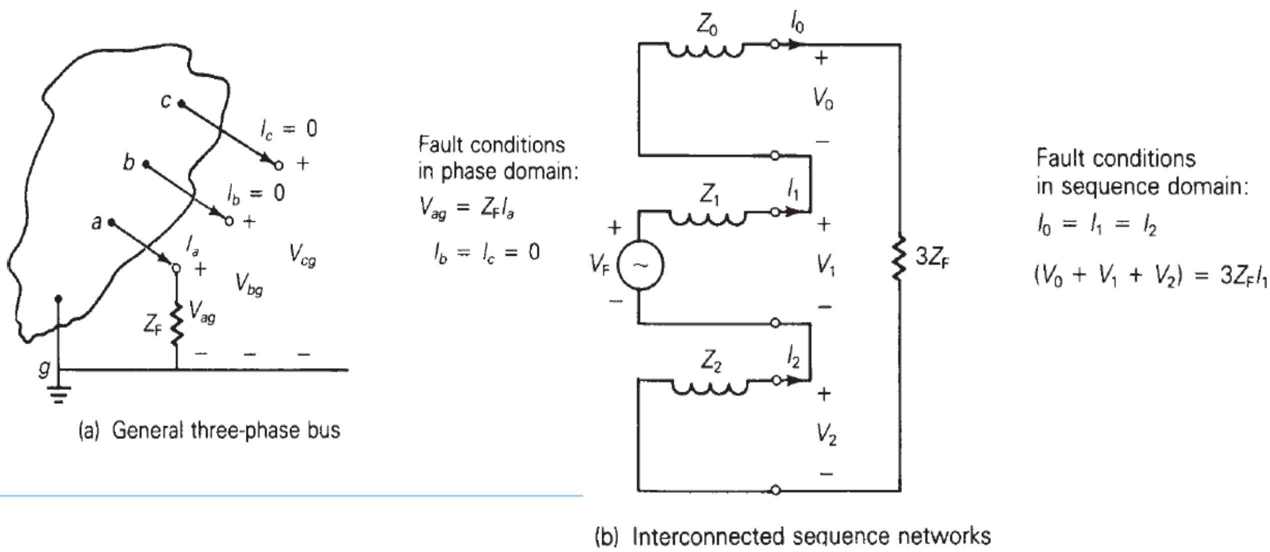
$$\begin{bmatrix} I''_a \\ I''_b \\ I''_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j7.558 \\ 0 \end{bmatrix} = \begin{bmatrix} 7.558\angle{-90^\circ} \\ 7.558\angle{150^\circ} \\ 7.558\angle{30^\circ} \end{bmatrix} \quad \text{per unit}$$ ■

* The sequence components of line-to-ground voltages at the faults terminals:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ V_F \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

9.2 Single line-to-ground faults:

FIGURE 9.7 Single line-to-ground fault



* Fault currents (sequence components):

$$I_0 = I_1 = I_2 = \frac{V_F}{Z_0 + Z_1 + Z_2 + (3Z_F)}$$

* Fault currents (phase domain):

$$I_a = I_0 + I_1 + I_2 = 3I_1 = \frac{3V_F}{Z_0 + Z_1 + Z_2 + (3Z_F)}$$, $I_b = I_c = 0$

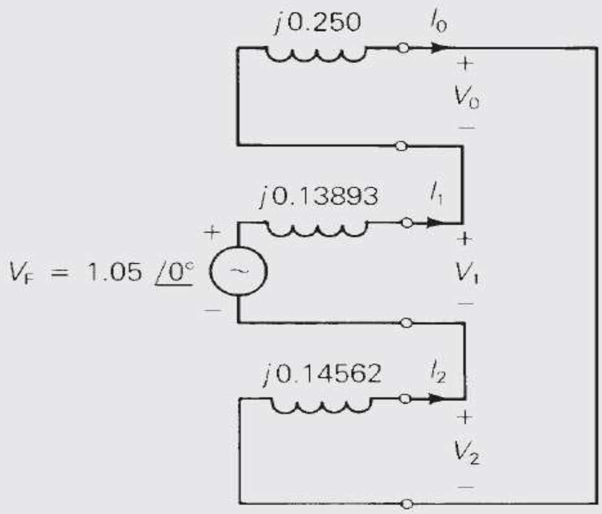
EXAMPLE 9.3 **Single line-to-ground short-circuit calculations using sequence networks**

Calculate the subtransient fault current in per-unit and in kA for a bolted single line-to-ground short circuit from phase a to ground at bus 2 in Example 9.1. Also calculate the per-unit line-to-ground voltages at faulted bus 2.

SOLUTION The zero-, positive-, and negative-sequence networks in Figure 9.5 are connected in series at the fault terminals, as shown in Figure 9.8.

FIGURE 9.8

Example 9.3: Single line-to-ground fault at bus 2



Since the short circuit is bolted, $Z_F = 0$. From (9.2.7), the sequence currents are:

$$I_0 = I_1 = I_2 = \frac{1.05\underline{/0^\circ}}{j(0.25 + 0.13893 + 0.14562)}$$

$$= \frac{1.05}{j0.53455} = -j1.96427 \quad \text{per unit}$$

From (9.2.8), the subtransient fault current is

$$I_a'' = 3(-j1.96427) = -j5.8928 \quad \text{per unit}$$

The base current at bus 2 is $100/(13.8\sqrt{3}) = 4.1837$ kA. Therefore,

$$I_a'' = (-j5.8928)(4.1837) = 24.65\underline{/-90^\circ} \quad \text{kA}$$

From (9.1.1), the sequence components of the voltages at the fault are

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.05\underline{/0^\circ} \\ 0 \end{bmatrix} - \begin{bmatrix} j0.25 & 0 & 0 \\ 0 & j0.13893 & 0 \\ 0 & 0 & j0.14562 \end{bmatrix} \begin{bmatrix} -j1.96427 \\ -j1.96427 \\ -j1.96427 \end{bmatrix}$$

$$= \begin{bmatrix} -0.49107 \\ 0.77710 \\ -0.28604 \end{bmatrix} \quad \text{per unit}$$

Transforming to the phase domain, the line-to-ground voltages at faulted bus 2 are

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.49107 \\ 0.77710 \\ -0.28604 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.179\underline{/231.3^\circ} \\ 1.179\underline{/128.7^\circ} \end{bmatrix} \quad \text{per unit}$$

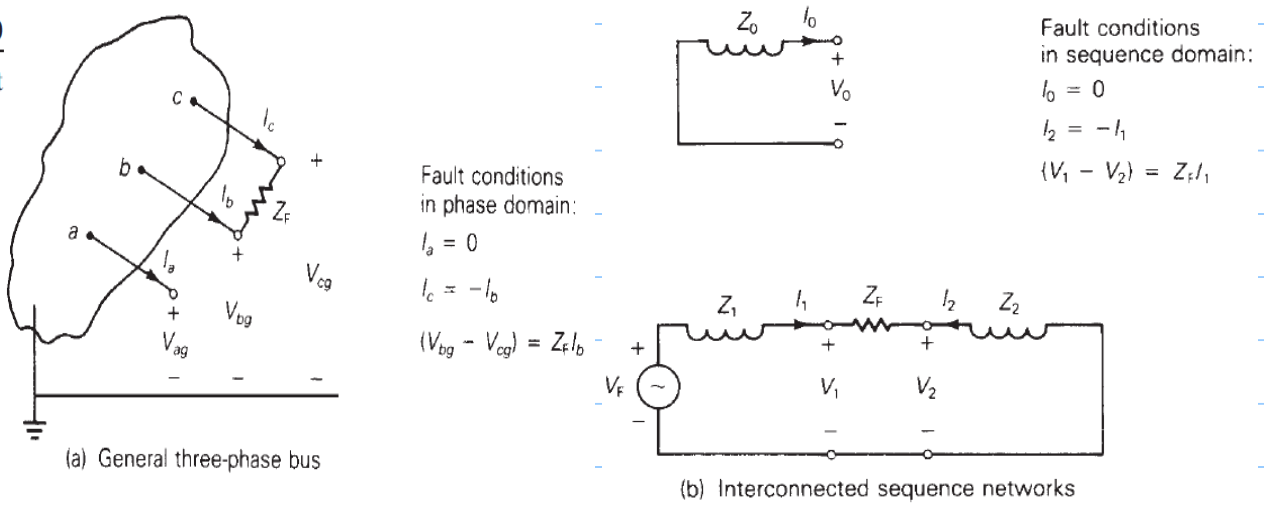
Note that $V_{ag} = 0$, as specified by the fault conditions. Also $I_b'' = I_c'' = 0$.

(PowerWorld)

9.3 Line-to-line faults:

FIGURE 9.10

Line-to-line fault



* Fault currents (sequence components):

$$I_1 = -I_2 = \frac{V_F}{(Z_1 + Z_2 + Z_F)}, \quad I_0 = 0$$

* Fault currents (phase domain):

$$I_b = I_0 + a^2 I_1 + a I_2 = (a^2 - a) I_1$$

$$= -j\sqrt{3} I_1 = \frac{-j\sqrt{3} V_F}{(Z_1 + Z_2 + Z_F)}$$

$$I_a = I_0 + I_1 + I_2 = 0$$

and

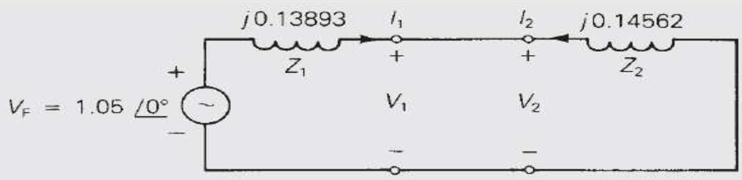
$$I_c = I_0 + a I_1 + a^2 I_2 = (a - a^2) I_1 = -I_b$$

EXAMPLE 9.4 Line-to-line short-circuit calculations using sequence networks

Calculate the subtransient fault current in per-unit and in kA for a bolted line-to-line fault from phase b to c at bus 2 in Example 9.1.

FIGURE 9.11

Example 9.4: Line-to-line fault at bus 2



SOLUTION The positive- and negative-sequence networks in Figure 9.5 are connected in parallel at the fault terminals, as shown in Figure 9.11. From (9.3.10) with $Z_F = 0$, the sequence fault currents are

$$I_1 = -I_2 = \frac{1.05\angle 0^\circ}{j(0.13893 + 0.14562)} = 3.690\angle -90^\circ$$

$$I_0 = 0$$

From (9.3.11), the subtransient fault current in phase b is

$$I_b'' = (-j\sqrt{3})(3.690\angle -90^\circ) = -6.391 = 6.391\angle 180^\circ \quad \text{per unit}$$

Using 4.1837 kA as the base current at bus 2,

$$I_b'' = (6.391\angle 180^\circ)(4.1837) = 26.74\angle 180^\circ \quad \text{kA}$$

Using 4.1837 kA as the base current at bus 2,

$$I_b'' = (6.391\underline{/180^\circ})(4.1837) = 26.74\underline{/180^\circ} \quad \text{kA}$$

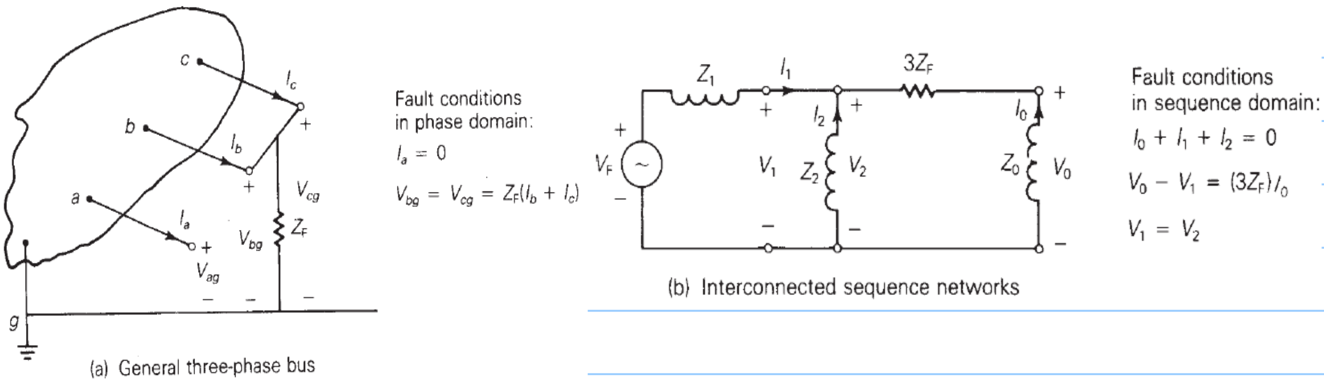
Also, from (9.3.12) and (9.3.13),

$$I_a'' = 0 \qquad I_c'' = 26.74\underline{/0^\circ} \quad \text{kA}$$

The line-to-line fault results for this example can be shown in PowerWorld Simulator by repeating the Example 9.3 procedure, with the exception that the Fault Type field value should be "Line-to-Line." ■

9.4 Double line-to-ground fault:

FIGURE 9.12 Double line-to-ground fault



* Fault currents (sequence components):

$$I_1 = \frac{V_F}{Z_1 + [Z_2/\!/(Z_0 + 3Z_F)]} = \frac{V_F}{Z_1 + \left[\dfrac{Z_2(Z_0 + 3Z_F)}{Z_2 + Z_0 + 3Z_F}\right]}$$

$$I_2 = (-I_1)\left(\frac{Z_0 + 3Z_F}{Z_0 + 3Z_F + Z_2}\right)$$

$$I_0 = (-I_1)\left(\frac{Z_2}{Z_0 + 3Z_F + Z_2}\right)$$

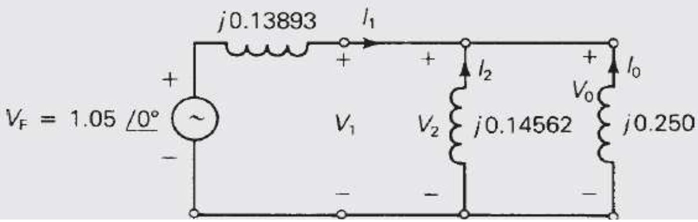
EXAMPLE 9.5 Double line-to-ground short-circuit calculations using sequence networks

Calculate (a) the subtransient fault current in each phase, (b) neutral fault current, and (c) contributions to the fault current from the motor and from the transmission line, for a bolted double line-to-ground fault from phase b to c to ground at bus 2 in Example 9.1. Neglect the Δ–Y transformer phase shifts.

SOLUTION

a. The zero-, positive-, and negative-sequence networks in Figure 9.5 are connected in parallel at the fault terminals in Figure 9.13. From (9.4.12) with $Z_F = 0$,

FIGURE 9.13 Example 9.5: Double line-to-ground fault at bus 2



$$I_1 = \frac{1.05\underline{/0^\circ}}{j\left[0.13893 + \frac{(0.14562)(0.25)}{0.14562 + 0.25}\right]} = \frac{1.05\underline{/0^\circ}}{j0.23095}$$

$$= -j4.5464 \quad \text{per unit}$$

From (9.4.13) and (9.4.14),

$$I_2 = (+j4.5464)\left(\frac{0.25}{0.25 + 0.14562}\right) = j2.8730 \quad \text{per unit}$$

$$I_0 = (+j4.5464)\left(\frac{0.14562}{0.25 + 0.14562}\right) = j1.6734 \quad \text{per unit}$$

Transforming to the phase domain, the subtransient fault currents are:

$$\begin{bmatrix} I''_a \\ I''_b \\ I''_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} +j1.6734 \\ -j4.5464 \\ +j2.8730 \end{bmatrix} = \begin{bmatrix} 0 \\ 6.8983\underline{/158.66^\circ} \\ 6.8983\underline{/21.34^\circ} \end{bmatrix} \quad \text{per unit}$$

Using the base current of 4.1837 kA at bus 2,

$$\begin{bmatrix} I''_a \\ I''_b \\ I''_c \end{bmatrix} = \begin{bmatrix} 0 \\ 6.8983\underline{/158.66^\circ} \\ 6.8983\underline{/21.34^\circ} \end{bmatrix}(4.1837) = \begin{bmatrix} 0 \\ 28.86\underline{/158.66^\circ} \\ 28.86\underline{/21.34^\circ} \end{bmatrix} \quad \text{kA}$$

b. The neutral fault current is

$$I_n = (I''_b + I''_c) = 3I_0 = j5.0202 \quad \text{per unit}$$

$$= (j5.0202)(4.1837) = 21.00\underline{/90^\circ} \quad \text{kA}$$

c. Neglecting Δ–Y transformer phase shifts, the contributions to the fault current from the motor and transmission line can be obtained from Figure 9.4. From the zero-sequence network, Figure 9.4(a), the contribution to the zero-sequence fault current from the line is zero, due to the transformer connection. That is,

$$I_{\text{line }0} = 0$$

$$I_{\text{motor }0} = I_0 = j1.6734 \quad \text{per unit}$$

From the positive-sequence network, Figure 9.4(b), the positive terminals of the internal machine voltages can be connected, since $E''_g = E''_m$. Then, by current division,

$$I_{\text{line }1} = \frac{X''_m}{X''_m + (X''_g + X_{T1} + X_{\text{line }1} + X_{T2})} I_1$$

$$= \frac{0.20}{0.20 + (0.455)}(-j4.5464) = -j1.3882 \quad \text{per unit}$$

$$I_{\text{motor }1} = \frac{0.455}{0.20 + 0.455}(-j4.5464) = -j3.1582 \quad \text{per unit}$$

From the negative-sequence network, Figure 9.4(c), using current division,

$$I_{\text{line }2} = \frac{0.21}{0.21 + 0.475}(j2.8730) = j0.8808 \quad \text{per unit}$$

$$I_{\text{motor }2} = \frac{0.475}{0.21 + 0.475}(j2.8730) = j1.9922 \quad \text{per unit}$$

Transforming to the phase domain with base currents of 0.41837 kA for the line and 4.1837 kA for the motor,

$$\begin{bmatrix} I''_{\text{line }a} \\ I''_{\text{line }b} \\ I''_{\text{line }c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j1.3882 \\ j0.8808 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5074\underline{/-90^\circ} \\ 1.9813\underline{/172.643^\circ} \\ 1.9813\underline{/7.357^\circ} \end{bmatrix} \quad \text{per unit}$$

$$= \begin{bmatrix} 0.2123\underline{/-90^\circ} \\ 0.8289\underline{/172.643^\circ} \\ 0.8289\underline{/7.357^\circ} \end{bmatrix} \quad \text{kA}$$

$$\begin{bmatrix} I''_{\text{motor } a} \\ I''_{\text{motor } b} \\ I''_{\text{motor } c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j1.6734 \\ -j3.1582 \\ j1.9922 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5074\underline{/90^\circ} \\ 4.9986\underline{/153.17^\circ} \\ 4.9986\underline{/26.83^\circ} \end{bmatrix} \quad \text{per unit}$$

$$= \begin{bmatrix} 2.123\underline{/90^\circ} \\ 20.91\underline{/153.17^\circ} \\ 20.91\underline{/26.83^\circ} \end{bmatrix} \quad \text{kA}$$

EXAMPLE 9.6 Effect of Δ–Y transformer phase shift on fault currents

Rework Example 9.5, with the Δ–Y transformer phase shifts included. Assume American standard phase shift.

SOLUTION The sequence networks of Figure 9.4 are redrawn in Figure 9.14 with ideal phase-shifting transformers representing Δ–Y phase shifts. In

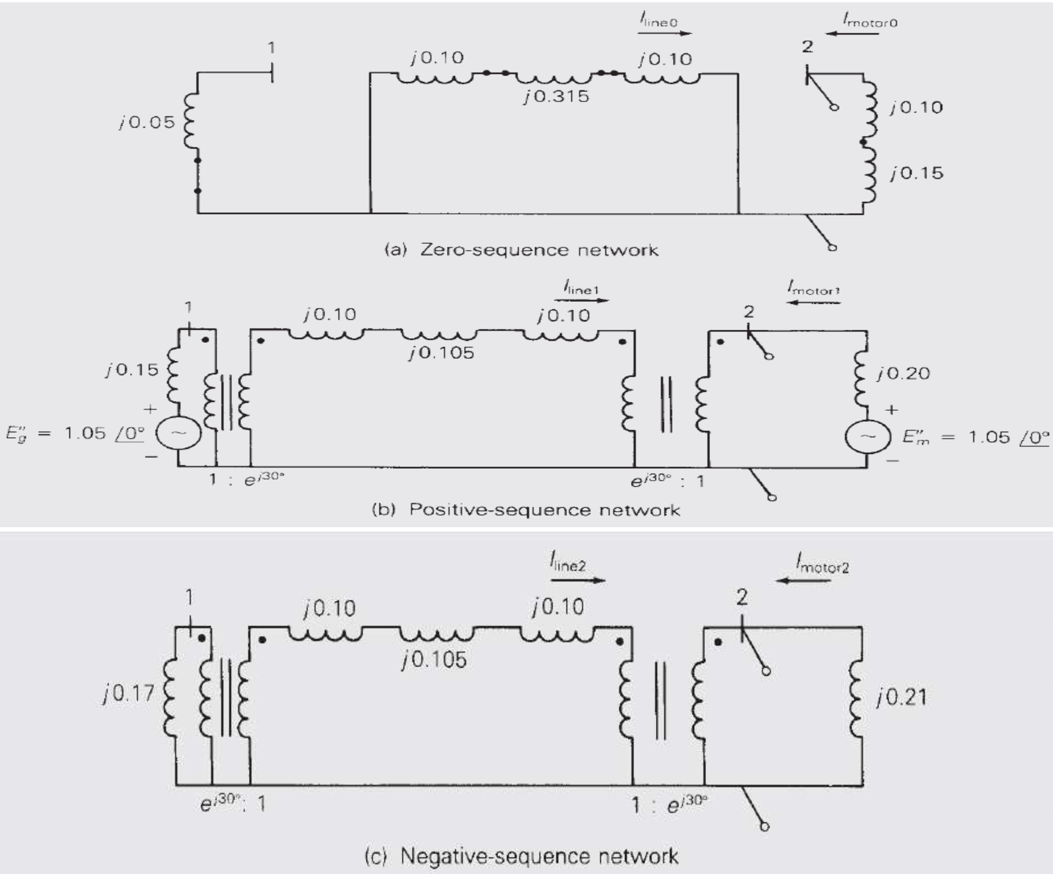


FIGURE 9.14 Sequence networks for Example 9.6

accordance with the American standard, positive-sequence quantities on the high-voltage side of the transformers lead their corresponding quantities on the low-voltage side by 30°. Also, the negative-sequence phase shifts are the reverse of the positive-sequence phase shifts.

a. Recall from Section 3.1 and (3.1.26) that per-unit impedance is unchanged when it is referred from one side of an ideal phase-shifting transformer to the other. Accordingly, the Thévenin equivalents of the sequence networks in Figure 9.14, as viewed from fault bus 2, are the same as those given in Figure 9.5. Therefore, the sequence components as well as the phase components of the fault currents are the same as those given in Example 9.5(a).

b. The neutral fault current is the same as that given in Example 9.5(b).

c. The zero-sequence network, Figure 9.14(a), is the same as that given in Figure 9.4(a). Therefore, the contributions to the zero-sequence fault current from the line and motor are the same as those given in Example 9.5(c).

$$I_{\text{line }0} = 0 \qquad I_{\text{motor }0} = I_0 = j1.6734 \quad \text{per unit}$$

The contribution to the positive-sequence fault current from the line in Figure 9.13(b) leads that in Figure 9.4(b) by 30°. That is,

$$I_{\text{line }1} = (-j1.3882)(1\underline{/30^\circ}) = 1.3882\underline{/-60^\circ} \quad \text{per unit}$$

$$I_{\text{motor }1} = -j3.1582 \quad \text{per unit}$$

Similarly, the contribution to the negative-sequence fault current from the line in Figure 9.14(c) lags that in Figure 9.4(c) by 30°. That is,

$$I_{\text{line }2} = (j0.8808)(1\underline{/-30^\circ}) = 0.8808\underline{/60^\circ} \quad \text{per unit}$$

$$I_{\text{motor }2} = j1.9922 \quad \text{per unit}$$

Thus, the sequence currents as well as the phase currents from the motor are the same as those given in Example 9.5(c). Also, the sequence currents from the line have the same magnitudes as those given in Example 9.5(c), but the positive- and negative-sequence line currents are shifted by +30° and −30°, respectively. Transforming the line currents to the phase domain:

$$\begin{bmatrix} I''_{\text{line }a} \\ I''_{\text{line }b} \\ I''_{\text{line }c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 1.3882\underline{/-60^\circ} \\ 0.8808\underline{/60^\circ} \end{bmatrix}$$

$$= \begin{bmatrix} 1.2166/-21.17^\circ \\ 2.2690\underline{/180^\circ} \\ 1.2166\underline{/21.17^\circ} \end{bmatrix} \quad \text{per unit}$$

$$= \begin{bmatrix} 0.5090\underline{/-21.17^\circ} \\ 0.9492\underline{/180^\circ} \\ 0.5090\underline{/21.17^\circ} \end{bmatrix} \quad \text{kA}$$

In conclusion, Δ–Y transformer phase shifts have no effect on the fault currents and no effect on the contribution to the fault currents on the fault side of the Δ–Y transformers. However, on the other side of the Δ–Y transformers, the positive- and negative-sequence components of the contributions to the fault currents are shifted by ±30°, which affects both the magnitude as well as the angle of the phase components of these fault contributions for unsymmetrical faults. ■

* Summary of faults:

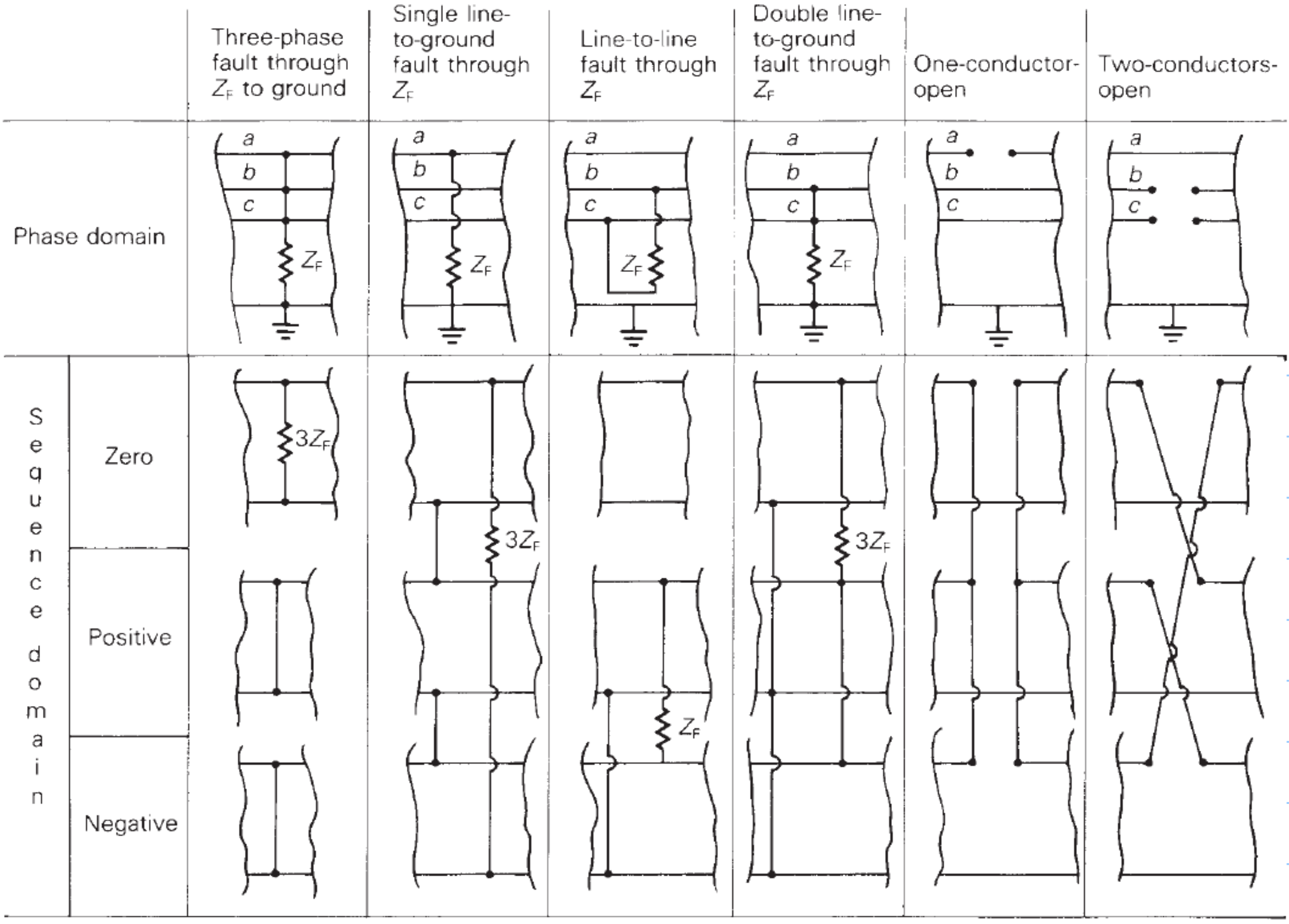


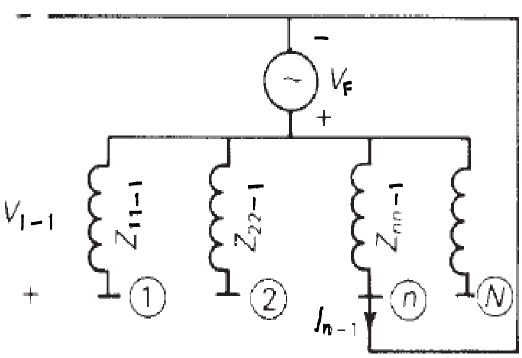
FIGURE 9.15 Summary of faults

9.5 Sequence bus impedance matrix:

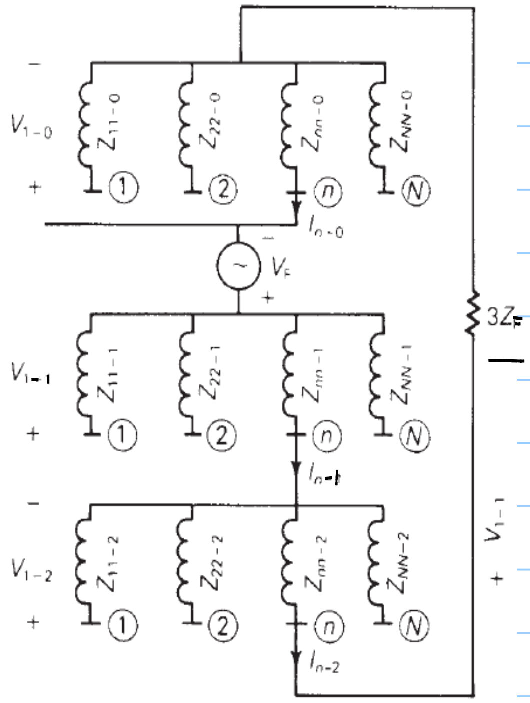
* Rake equivalent sequence network: (Fault at bus n)

Assumptions:

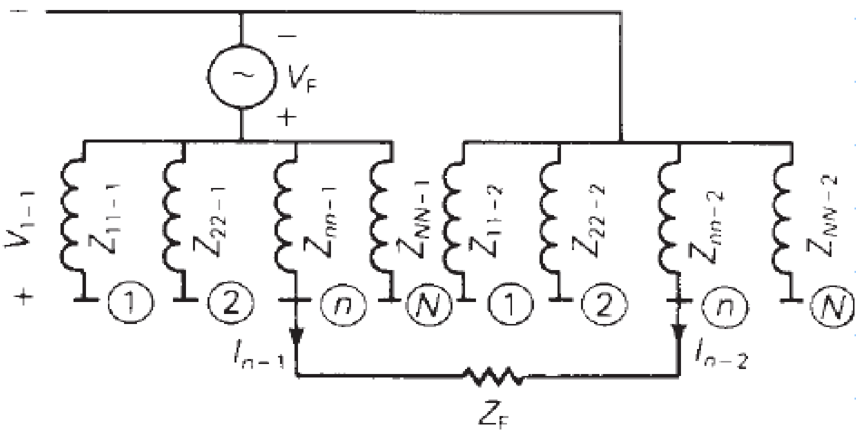
For simplicity, resistances, shunt admittances, nonrotating impedance loads, and prefault load currents are neglegted.



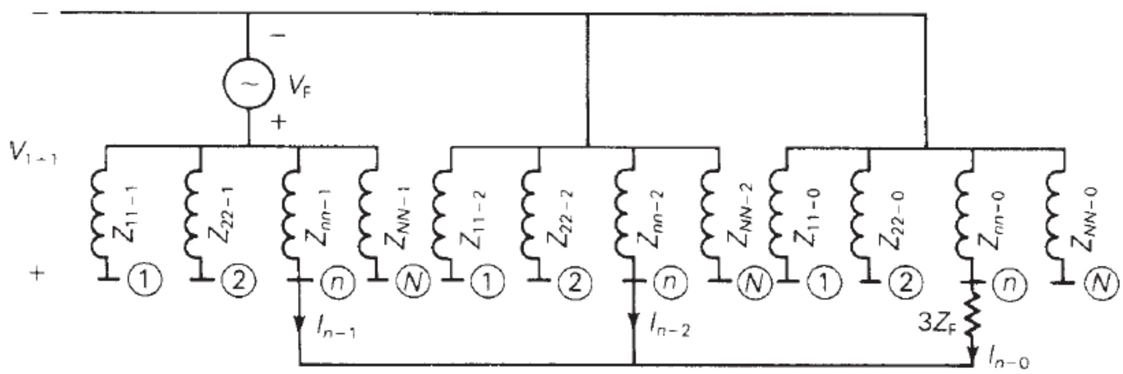
(a) Three-phase fault



(b) Single line-to-ground fault



(c) Line-to-line fault



(d) Double line-to-ground fault

Balanced three-phase fault:

$$I_{n-1} = \frac{V_F}{Z_{nn-1}} \tag{9.5.1}$$

$$I_{n-0} = I_{n-2} = 0 \tag{9.5.2}$$

Single line-to-ground fault (phase *a* to ground):

$$I_{n-0} = I_{n-1} = I_{n-2} = \frac{V_F}{Z_{nn-0} + Z_{nn-1} + Z_{nn-2} + 3Z_F} \tag{9.5.3}$$

Line-to-line fault (phase *b* to *c*):

$$I_{n-1} = -I_{n-2} = \frac{V_F}{Z_{nn-1} + Z_{nn-2} + Z_F} \tag{9.5.4}$$

$$I_{n-0} = 0 \tag{9.5.5}$$

Double line-to-ground fault (phase *b* to *c* to ground):

$$I_{n-1} = \frac{V_F}{Z_{nn-1} + \left[\frac{Z_{nn-2}(Z_{nn-0} + 3Z_F)}{Z_{nn-2} + Z_{nn-0} + 3Z_F}\right]} \tag{9.5.6}$$

$$I_{n-2} = (-I_{n-1})\left(\frac{Z_{nn-0} + 3Z_F}{Z_{nn-0} + 3Z_F + Z_{nn-2}}\right) \tag{9.5.7}$$

$$I_{n-0} = (-I_{n-1})\left(\frac{Z_{nn-2}}{Z_{nn-0} + 3Z_F + Z_{nn-2}}\right) \tag{9.5.8}$$

Also from Figure 9.16, the sequence components of the line-to-ground voltages at any bus k during a fault at bus n are:

$$\begin{bmatrix} V_{k-0} \\ V_{k-1} \\ V_{k-2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_F \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{kn-0} & 0 & 0 \\ 0 & Z_{kn-1} & 0 \\ 0 & 0 & Z_{kn-2} \end{bmatrix} \begin{bmatrix} I_{n-0} \\ I_{n-1} \\ I_{n-2} \end{bmatrix} \tag{9.5.9}$$

EXAMPLE 9.7 Single line-to-ground short-circuit calculations using $Z_{bus\,0}$, $Z_{bus\,1}$, and $Z_{bus\,2}$

Faults at buses 1 and 2 for the three-phase power system given in Example 9.1 are of interest. The prefault voltage is 1.05 per unit. Prefault load current is neglected. (a) Determine the per-unit zero-, positive-, and negative-sequence bus impedance matrices. Find the subtransient fault current in per-unit for a bolted single line-to-ground fault current from phase a to ground (b) at bus 1 and (c) at bus 2. Find the per-unit line-to-ground voltages at (d) bus 1 and (e) bus 2 during the single line-to-ground fault at bus 1.

SOLUTION

a. Referring to Figure 9.4(a), the zero-sequence bus admittance matrix is

$$\boldsymbol{Y}_{bus\,0} = -j\left[\begin{array}{c|c} 20 & 0 \\ \hline 0 & 4 \end{array}\right] \quad \text{per unit}$$

Inverting $\boldsymbol{Y}_{bus\,0}$,

$$\boldsymbol{Z}_{bus\,0} = j\left[\begin{array}{c|c} 0.05 & 0 \\ \hline 0 & 0.25 \end{array}\right] \quad \text{per unit}$$

Note that the transformer leakage reactances and the zero-sequence transmission-line reactance in Figure 9.4(a) have no effect on $\boldsymbol{Z}_{bus\,0}$. The transformer Δ connections block the flow of zero-sequence current from the transformers to bus 1 and 2.

The positive-sequence bus admittance matrix, from Figure 9.4(b), is

$$\boldsymbol{Y}_{\text{bus 1}} = -j\left[\begin{array}{c|c} 9.9454 & -3.2787 \\ \hline -3.2787 & 8.2787 \end{array}\right] \quad \text{per unit}$$

Inverting $\boldsymbol{Y}_{\text{bus 1}}$,

$$\boldsymbol{Z}_{\text{bus 1}} = j\left[\begin{array}{c|c} 0.11565 & 0.04580 \\ \hline 0.04580 & 0.13893 \end{array}\right] \quad \text{per unit}$$

Similarly, from Figure 9.4(c)

$$\boldsymbol{Y}_{\text{bus 2}} = -j\left[\begin{array}{c|c} 9.1611 & -3.2787 \\ \hline -3.2787 & 8.0406 \end{array}\right]$$

Inverting $\boldsymbol{Y}_{\text{bus 2}}$,

$$\boldsymbol{Z}_{\text{bus 2}} = j\left[\begin{array}{c|c} 0.12781 & 0.05212 \\ \hline 0.05212 & 0.14562 \end{array}\right] \quad \text{per unit}$$

b. From (9.5.3), with $n = 1$ and $Z_F = 0$, the sequence fault currents are

$$I_{1-0} = I_{1-1} = I_{1-2} = \frac{V_F}{Z_{11-0} + Z_{11-1} + Z_{11-2}}$$

$$= \frac{1.05\underline{/0^\circ}}{j(0.05 + 0.11565 + 0.12781)} = \frac{1.05}{j0.29346} = -j3.578 \quad \text{per unit}$$

The subtransient fault currents at bus 1 are, from (8.1.16),

$$\begin{bmatrix} I''_{1a} \\ I''_{1b} \\ I''_{1c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} -j3.578 \\ -j3.578 \\ -j3.578 \end{bmatrix} = \begin{bmatrix} -j10.73 \\ 0 \\ 0 \end{bmatrix} \quad \text{per unit}$$

c. Again from (9.5.3), with $n = 2$ and $Z_F = 0$,

$$I_{2-0} = I_{2-1} = I_{2-2} = \frac{V_F}{Z_{22-0} + Z_{22-1} + Z_{22-2}}$$

$$= \frac{1.05\underline{/0^\circ}}{j(0.25 + 0.13893 + 0.14562)} = \frac{1.05}{j0.53455}$$

$$= -j1.96427 \quad \text{per unit}$$

and

$$\begin{bmatrix} I''_{2a} \\ I''_{2b} \\ I''_{2c} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} -j1.96427 \\ -j1.96427 \\ -j1.96427 \end{bmatrix} = \begin{bmatrix} -j5.8928 \\ 0 \\ 0 \end{bmatrix} \quad \text{per unit}$$

This is the same result as obtained in Example 9.3.

d. The sequence components of the line-to-ground voltages at bus 1 during the fault at bus 1 are, from (9.5.9), with $k = 1$ and $n = 1$,

$$\begin{bmatrix} V_{1-0} \\ V_{1-1} \\ V_{1-2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.05\underline{/0^\circ} \\ 0 \end{bmatrix} - \begin{bmatrix} j0.05 & 0 & 0 \\ 0 & j0.11565 & 0 \\ 0 & 0 & j0.12781 \end{bmatrix}\begin{bmatrix} -j3.578 \\ -j3.578 \\ -j3.578 \end{bmatrix}$$

$$= \begin{bmatrix} -0.1789 \\ 0.6362 \\ -0.4573 \end{bmatrix} \quad \text{per unit}$$

and the line-to-ground voltages at bus 1 during the fault at bus 1 are

$$\begin{bmatrix} V_{1-ag} \\ V_{1-bg} \\ V_{1-cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.1789 \\ +0.6362 \\ -0.4573 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.9843\underline{/254.2^\circ} \\ 0.9843\underline{/105.8^\circ} \end{bmatrix} \quad \text{per unit}$$

e. The sequence components of the line-to-ground voltages at bus 2 during the fault at bus 1 are, from (9.5.9), with $k = 2$ and $n = 1$,

$$\begin{bmatrix} V_{2-0} \\ V_{2-1} \\ V_{2-2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.05\underline{/0^\circ} \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & j0.04580 & 0 \\ 0 & 0 & j0.05212 \end{bmatrix} \begin{bmatrix} -j3.578 \\ -j3.578 \\ -j3.578 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.8861 \\ -0.18649 \end{bmatrix} \quad \text{per unit}$$

Note that since both bus 1 and 2 are on the low-voltage side of the Δ–Y transformers in Figure 9.3, there is no shift in the phase angles of these sequence voltages. From the above, the line-to-ground voltages at bus 2 during the fault at bus 1 are

$$\begin{bmatrix} V_{2-ag} \\ V_{2-bg} \\ V_{2-cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.8861 \\ -0.18649 \end{bmatrix}$$

$$= \begin{bmatrix} 0.70 \\ 0.9926\underline{/249.4^\circ} \\ 0.9926\underline{/110.6^\circ} \end{bmatrix} \quad \text{per unit}$$

■

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